Sorting Does Not Homogenize:
Fractionalization in American Parties

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Abstract

American partisan coalitions are increasingly sorted: Democrats and Republicans have become more distinct in terms of race, religion, and ideology. But are the two parties also becoming more internally homogeneous on these dimensions? We show that between-party sorting and within-party fractionalization can happen simultaneously. Using popular measures of fractionalization and sorting, we provide a mathematical illustration of this possibility. Then, we use time-series data from the American National Election Studies to trace the social compositions of the two parties from 1972 to 2024. We show that sorting and fractionalization—especially on race and religion—indeed happened in parallel and that this was true for both Democrats and Republicans. Our results help to explain negative partisanship and why outparty stereotype corrections have been found to reduce affective polarization. More broadly, our findings call for increased attention to social dynamics within, not just between, partisan coalitions.

Keywords: fractionalization, partisanship, polarization, sorting

Partisan social sorting is among the most important trends in modern American politics: the Democratic and Republican coalitions are increasingly distinct from one another in terms of race, religion, and ideology—and this fuels affective polarization (Abramowitz 2011; Mason 2018; Zingher 2018; Zhirkov and Valentino 2022). But do these growing differences between parties necessitate increased homogeneity within parties?

Our contribution starts from a proposition that between-party social sorting and within-party social fractionalization can happen simultaneously. Sorting occurs when two parties become more demographically distinct. Fractionalization occurs when a party becomes more evenly divided (less homogeneous) on a given dimension. Although in the context of a two-party system, these may seem like mutually exclusive possibilities, they can co-occur if overall diversity increases within a society or if voters have an option to not identify with one of the two parties (i.e., political independence).

In this paper, we demonstrate how social fractionalization within the U.S. parties happened in parallel with social sorting between the parties over the last five decades. We start with a brief overview of sorting, demographic change, and the demographic compositions of American party coalitions. Then, using the most popular measures of sorting and fractionalization, we provide a simple mathematical illustration of how sorting and fractionalization can co-occur. We proceed to show how this possibility was realized in American partisan politics. Using time-series data from the American National Election Studies (ANES), we trace the compositions of the two partisan coalitions from 1972 to 2024 and show that growing between-party sorting indeed coexisted with growing within-party fractionalization over the studied period.

Our results also highlight important differences in how the two partisan coalitions changed in terms of race and religion on one hand and ideology on the other hand. Over the studied five decades, both parties have become less white and less Christian—but these changes were much faster in the Democratic Party than in the Republican Party thus

producing the racial and religious gap. Ideologically, Democrats have moved from being a predominantly moderate party to one with equal shares of moderates and liberals. Republicans have experienced an opposite evolution: from an ideologically divided party into a coalition clearly dominated by conservatives. We conclude by discussing the implications of our findings for political polarization and the modern U.S. party system.

U.S. Parties: Sorting Between, Fractionalization Within?

Although Democrats and Republicans have sorted by ideology, they are not ideologically monolithic. Within congressional parties, data show evidence of distinct ideological factions with their own donor bases and unique voting patterns (Clarke 2020). On the mass level, research shows evidence of affective polarization between the moderate and more ideologically extreme wings within both parties (Groenendyk et al. 2020). Diversity is even more prevalent on other dimensions: the two parties—and the Democrats in particular—remain divided across group attitudes (King 2022), social identities (Perez et al. 2022; Townsend et al. 2025), and psychological orientations (Wronski et al. 2018). While parties have become important social identity groups in their own right (Green et al. 2002; Huddy et al. 2015), they remain coalitions rooted in group sentiments (Kane et al. 2021).

Over the last few decades, American society has also experienced a major demographic transformation. Immigration has consistently increased racial diversity in the country, while the share of whites in the U.S. population continues to decrease (Frey 2015). Once overwhelmingly Christian, American society has also rapidly secularized as a growing number of Americans now identify as non-religious (Voas and Chaves 2016). Reactions to these trends have played a major role in reshaping party coalitions (Abrajano and Hajnal 2015). During the same period, many individuals exited both parties in favor of independence (Klar and Krupnikov 2016; Jones 2026).

These two processes, partisan sorting and demographic change, profoundly shape modern U.S. politics. Americans have sorted between parties based on race, religion, and

ideology, but if these changes are outpaced by demographic shifts, it leads to within-party fractionalization. This has important theoretical implications: if parties are becoming more distinct without becoming more homogeneous, the images of the prototypical Democrat and Republican can become clearer without reducing identity complexity (Roccas and Brewer 2002). This may help to explain how outgroup animus can flourish in the absence of strong ingroup attachments (Mason et al. 2021), why negative partisanship seems to be on the rise (Abramowitz and Webster 2016; Groenendyk 2018), and why outparty stereotype corrections often reduce polarization (Ahler and Sood 2018; Druckman et al. 2022; Voelkel et al. 2024), while inparty identity manipulations do not (West and Iyengar 2022). In short, the coexistence of between-party sorting and within-party fractionalization can clarify the coalitional dynamics underlying partisan polarization and may help to resolve long-standing debates over the explosion of partisan antipathy and how to address it.

A Mathematical Illustration

Social sorting between parties and social fractionalization within parties can occur simultaneously. For this to happen, the population of partisans must change either via changes in the overall demographics of the population or through shifts from partisanship to independence and vice versa. In this section, we provide a mathematical demonstration of this possibility using popular measures of sorting and fractionalization.

For simplicity but without loss of generality, consider a society that has two parties, $p \in \{D, R\}$, and is divided into two social groups, $X \in \{0, 1\}$.[1] Also consider two time periods, $t \in \{0, 1\}$. Define $\pi_p^{(t)}$ as the share of party p belonging to group $X = 1$ at time t or equivalently, the probability that a randomly chosen member of party p at time t belongs to group $X = 1$:

$$\pi_p^{(t)} = \Pr(X = 1 \mid p, t). \tag{1}$$

[1] These are restrictive assumptions. Allowing more than two social groups and/or more than two parties makes the result easier to obtain.

A popular measure of between-party sorting at time t, $S^{(t)}$, is the difference in party-specific shares of group $X = 1$ at time t (Mason 2018):

$$S^{(t)} = \pi_D^{(t)} - \pi_R^{(t)}. \tag{2}$$

The resulting score can range from -1 (all members of party R belong to group $X = 1$ whereas there are no members of that group in party D) to 1 (all members of party D belong to group $X = 1$ whereas there are no members of that group in party R). Section A of the supplementary information (SI) demonstrates that in the case of two groups, this definition of sorting on a given identity dimension is equivalent to the difference-in-difference measure divided by two.

Following a rich literature in comparative politics and political economy, we define within-party fractionalization as the degree to which a partisan coalition is divided into distinct groups that are roughly similar in size (Alesina et al. 2003). It is usually calculated as one minus the sum of the squares of group shares, but with only two groups it can be re-expressed as two times the product of group shares (see Section B of the SI). We further multiply it by two so that the resulting score can range from zero (no fractionalization, all party members belong to the same group) to one (maximum fractionalization, the party is equally divided between the two groups). The resulting measure of within-party fractionalization at time t, $F_p^{(t)}$, is:

$$F_p^{(t)} = 4\pi_p^{(t)}(1 - \pi_p^{(t)}). \tag{3}$$

Fractionalization has a direct individual-level interpretation: for each party member, it increases the probability that a randomly chosen co-partisan will differ from them on the corresponding identity dimension. Section C of the SI established this result. In other words, higher fractionalization implies higher *atypicality* of an average partisan within their own party.

Section D of the SI formally proves that between-party sorting and within-party fractionalization can happen simultaneously. This means that parties can become more

Table 1. Co-occurrence of sorting and fractionalization: an example

	$t = 0$	$t = 1$	Change
Share of group $X = 1$ in party D, $\pi_R^{(t)}$	.10	.40	.30
Share of group $X = 1$ in party R, $\pi_D^{(t)}$	.10	.20	.10
Sorting between parties, $S^{(t)}$	.00	.20	.20
Fractionalization within party D, $F_R^{(t)}$	.36	.96	.60
Fractionalization within party R, $F_D^{(t)}$	.36	.64	.28

distinct from one another (higher sorting) even as the average partisan becomes less similar to other members of their own party (higher fractionalization).

Table 1 provides an illustration of possible changes in between-party sorting and within-party fractionalization across two time points. Two things happen simultaneously: First, the overall share of group $X = 1$ in the society grows due to demographic changes, so that both $\pi_D^{(t)}$ and $\pi_R^{(t)}$ increase. Second, there is partisan social sorting: members of group $X = 1$ join party D with much higher rates than party R, so that $\pi_D^{(t)}$ increases faster than $\pi_R^{(t)}$. We calculate between-party sorting and within-party fractionalization using Equation 2 and Equation 3 respectively. The resulting scores demonstrate that the party system as a whole becomes more sorted—but both party D and party R also become more internally fractionalized.

This section provides an important mathematical illustration that, under certain conditions, social sorting between parties and social fractionalization within parties can happen simultaneously. And the United States should satisfy these conditions due to the combination of demographic change and partisan sorting over the last 50 years. To test whether American parties have turned more internally fractionalized while the party system as a whole has became more sorted, we trace the changes in the social compositions of the Democratic Party and the Republican Party from 1972 to 2024.

Table 2. Analyzed identity dimensions

Identity dimension	Categories: analysis	Categories: ANES
Race	White	White non-Hispanic
	Nonwhite	Black, Hispanic, Other
Religion	Christian	Protestant, Roman Catholic
	Non-Christian	Jewish, Other
Social class	Working class	Average working, Upper working, Other working
	Midde class	Average middle, Upper middle, Other middle
Ideology	Liberal	Slightly liberal, Liberal, Extremely liberal
	Moderate	Moderate, Haven't thought much about it
	Conservative	Slightly conservative, Conservative, Extremely conservative

An Empirical Illustration

To trace the social compositions of U.S. partisan coalitions, we use data from the ANES time-series. We concentrate on the four key cleavages: race, religion, social class, and ideology. To ensure comparability across dimensions, we dichotomize the cleavage categories.[2] See Table 2 for the descriptions and coding of the corresponding variables. For ideology, we collapse moderates with conservatives in the Democratic coalition and moderates with liberals in the Republican coalition. Our analysis covers years from 1972 (when the ideology question is first asked) to 2024 (the most recent election) yielding the total of 22 time points. The results to follow include only self-identified partisans.

We start by illustrating the changes in compositions of the two parties on race, religion, social class, and ideology from 1972 to 2024. They are presented graphically in Figure 1. In 1972, the Democratic coalition had white, Christian, working class, and ideologically moderate majorities. By 2024, the share of whites in the Democratic Party decreased to being only a small majority, the share of non-religious people grew turning

[2] While this categorization may be oversimplifying some real-world social cleavages, it provides the most conservative test of the hypothesis regarding the co-occurrence of between-party sorting and within-party fractionalization.

Christians into a minority, the share of middle-class supporters became equal to those from the working class, and liberals slightly outnumbered moderates. The Republican coalition in 1972 was overwhelmingly white, Christian, and evenly divided between working and middle class as well as between moderates and conservatives. By 2024, the shares of whites and Christians within the Republican Party decreased, although these two groups remained majorities on the respective cleavage dimensions. Over the same period, the even split on class among Republicans turned into the prevalence of the middle class, while there emerged a clear dominance of ideological conservatives within the party.

In agreement with earlier studies, these changes show clear sorting on race, religion, and ideology: Republicans emerge as more white, Christian, and conservative than Democrats. At the same time, the data suggest that the two partisan coalitions have not been getting more homogenous. If anything, internal diversity seems to have increased, particularly among Democrats. To demonstrate these changes, we turn to the comparison of between-party sorting and within-party fractionalization. The scores are calculated using the formulas provided in Equation 2 and Equation 3 respectively. The changes from 1972 to 2024 are presented graphically in Figure 2. They show that increasing between-party sorting on race and religion indeed happened in parallel to within-party fractionalization on the same social cleavage dimensions. It is worth noting that racial and religious fractionalization—within both parties!—increased much faster over the studied period than sorting. The patterns are less clear for social class that shows almost no changes in either sorting or fractionalization. Ideological sorting increased very strongly, but ideological homogeneity increased only among Republicans, whereas Democrats became more ideologically fractionalized.

Table 3 presents estimated linear trends in between-party sorting and within-party fractionalization with the corresponding statistical tests. They show significant increases in sorting on race, religion, and ideology but no significant change in sorting on social class.

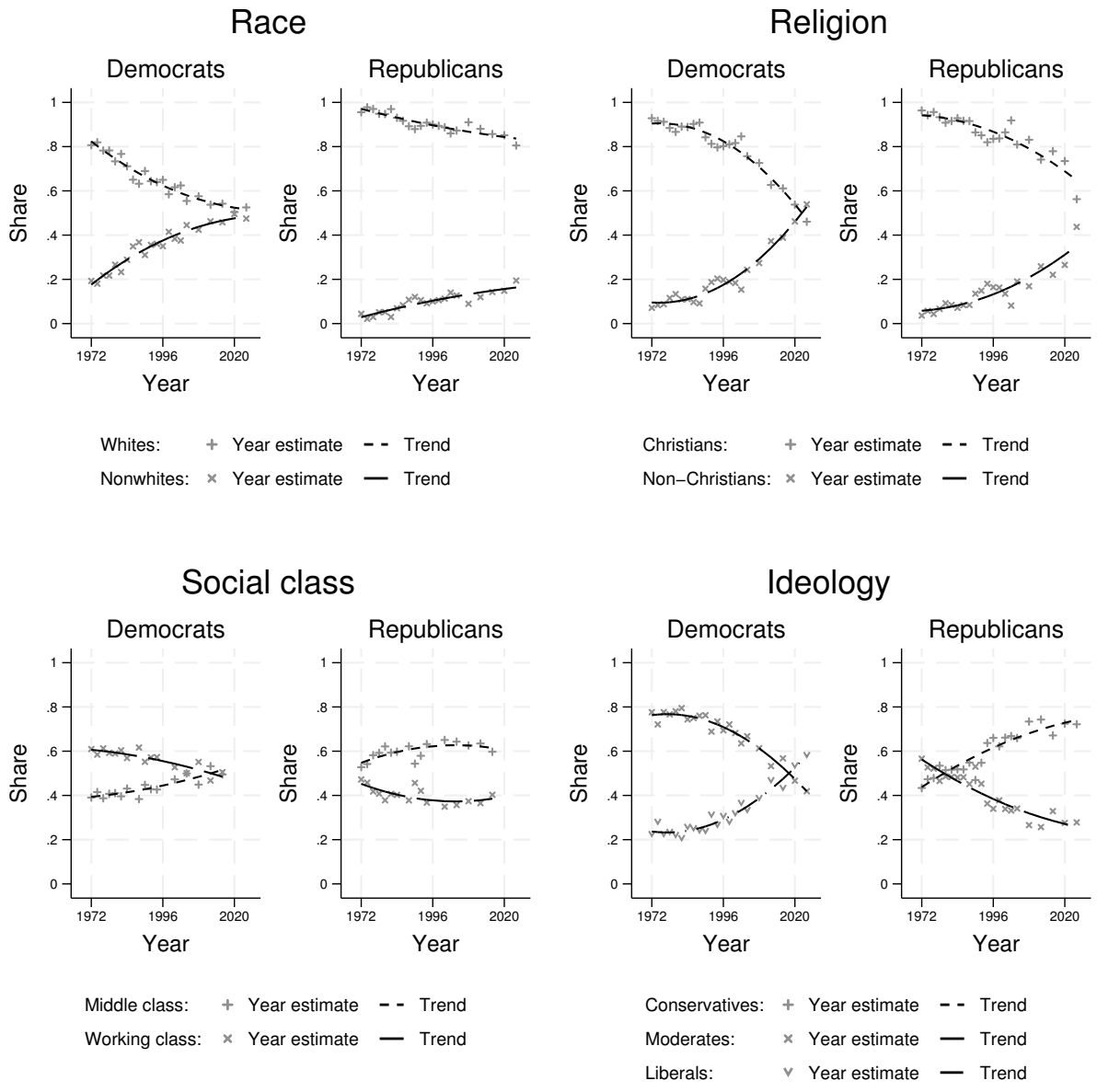

Figure 1. Changes in social compositions of the two parties, 1972–2024
Note. "Moderates" for Democrats include conservatives. "Moderates" for Republicans include liberals.

Fractionalization in the Democratic coalition was growing on all four dimensions. The Republican coalition is increasingly fractionalized on race and religion but increasingly

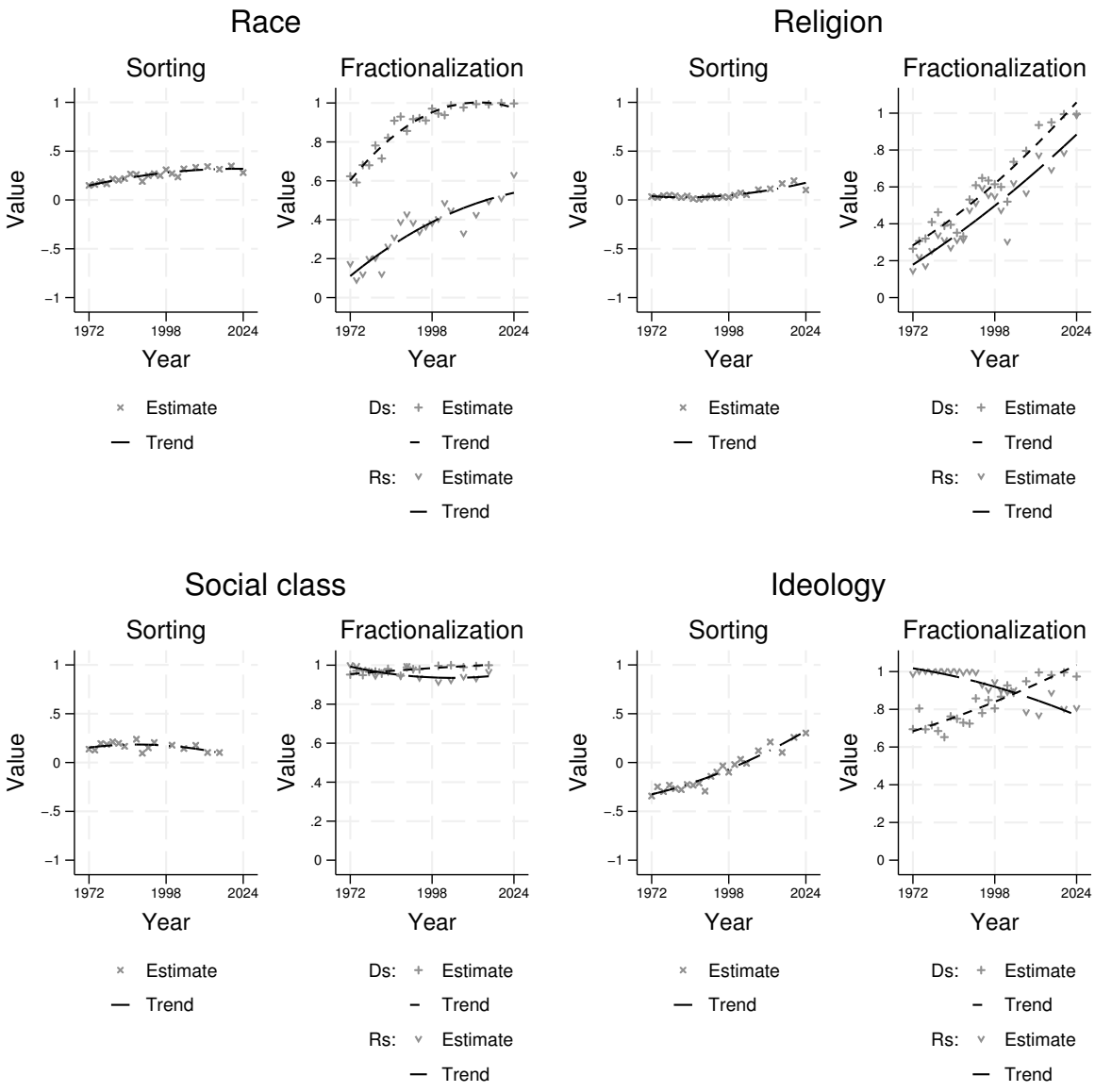

Figure 2. Changes in between-party sorting and within-party fractionalization, 1972–2024

homogenous on social class and ideology. Once again, these results demonstrate that between-party sorting and within-party fractionalization are not mutually exclusive.

Table 3. Estimated linear trends in sorting and fractionalization, 1972–2024

	Sorting between parties	Fractionalization, Democrats	Fractionalization, Republicans
Race	0.18***	0.39***	0.44***
	(0.02)	(0.05)	(0.05)
Religion	0.13***	0.77***	0.70***
	(0.02)	(0.06)	(0.07)
Social class	−0.06	0.06***	−0.06*
	(0.01)	(0.01)	(0.02)
Ideology	0.64***	0.35***	−0.25***
	(0.03)	(0.04)	(0.03)

Note. Standard errors in parentheses.
* $p < .05$, ** $p < .01$, *** $p < .001$

Conclusion

American parties are increasingly sorted, and the social profiles of the typical Democrat and the typical Republican are increasingly distinct (Mason 2018). But what is happening to social cleavages within the two parties? We have started by showing that between-party sorting and within-party fractionalization can happen simultaneously using two popular measures of these constructs. Then we have turned to the data from the ANES time-series to trace the compositions of the U.S. parties between 1972 and 2024. We demonstrate that between-party sorting happened in parallel to within-party fractionalization, especially on race and religion. Importantly, fractionalization is occurring in both parties. While it is true that U.S. parties are much more sorted now than in 1972, they are also more internally diverse political coalitions.

These findings highlight demographic trends within parties that are easy to overlook when researchers focus on differences between parties. For example, both party coalitions were predominantly white and Christian in 1972. The parties differ on these dimensions today, but not because whites and Christians have left the Democratic Party whereas the Republican Party has become more white and more Christian. Instead, the Republican

coalition has become *less* white and *less* Christian—it just has happened at a substantially slower rate than in the Democratic coalition and in the nation as a whole.

Even when it comes to ideological identification, where sorting has been the strongest over time, within party trends differ from what many readers may have expected. The Democratic coalition was once dominated by moderate identifiers and is now about evenly split between moderates and liberals. The Republican Party was once quite evenly split between moderate and conservative identifiers and is now dominated by conservatives. Thus, the ideological trends within the two parties are actually in opposite directions: the Republican coalition is more homogeneously conservative, whereas the Democratic coalition is more ideologically fractionalized than it was in 1972.

Our results also offer new insights on partisan polarization and how it may be overcome. First, while sorting has likely clarified the difference between the prototypical Republican and the prototypical Democrat, the prototypes are actually less representative in the sense that a given partisan is less likely to match their party's prototype than they were in the past. This helps to explain the rise of negative partisanship (Abramowitz and Webster 2016; Bankert 2021; Groenendyk 2012; Rothschild et al. 2021): since fractionalization has co-occurred with sorting, the average partisan has no reason to feel closer to the prototypical member of their own party, but they do have reason to feel farther from the prototypical member of the outparty.

Second, the coexistence of between-party sorting and within-party fractionalization helps to explain why studies have found stereotype corrections to be an effective method for reducing affective polarization and anti-democratic attitudes (Ahler and Sood 2018; Druckman et al. 2022; Voelkel et al. 2024). To correct outparty stereotypes is to correct the same inferential error that many scholars and journalists have been making: just because partisans have sorted on a given dimension does not mean parties have become more homogeneous on that dimension. Sorting has created sharper stereotypes while

fractionalization has made those stereotypes poorly representative of actual party compositions. It is the prototypical out-partisan toward whom Republicans and Democrats feel animosity. Thus, by making it clear that these prototypes are rather poor representation of the outparty, stereotype corrections help to reduce animosity.

While questions about the efficacy of partisan stereotype corrections have been raised recently (Dias et al. 2024), we believe that this is all the more reason to better understand the complexities underlying these stereotypes. As we have shown, when viewed from within versus between parties, the same trends can lead to different perspectives on party compositions, potentially affecting one's image of the prototypical Republican and the prototypical Democrat. Overall, divisions inside American partisan coalitions are clearly understudied: social and political processes happening *within* the two parties may have implications for the future of the U.S. party system no less important than the ones happening *between* them.

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Supplementary Information

Contents

Section A. Sorting measure equivalency

The sorting measure used in the paper is:

$$S^{(t)} = \pi_D^{(t)} - \pi_R^{(t)}. \tag{1}$$

Define the difference-in-difference measure of sorting and simpify:

$$\begin{aligned}
\tilde{S}^{(t)} &= (\pi_D^{(t)} - [1 - \pi_D^{(t)}]) - (\pi_R^{(t)} - [1 - \pi_R^{(t)}]) \\
&= (\pi_D^{(t)} - 1 + \pi_D^{(t)}) - (\pi_R^{(t)} - 1 + \pi_R^{(t)}) \\
&= (2\pi_D^{(t)} - 1) - (2\pi_R^{(t)} - 1) \\
&= 2\pi_D^{(t)} - 1 - 2\pi_R^{(t)} + 1 \\
&= 2\pi_D^{(t)} - 2\pi_R^{(t)} \\
&= 2\left(\pi_D^{(t)} - \pi_R^{(t)}\right)
\end{aligned}$$

So:

$$S^{(t)} = \frac{1}{2}\tilde{S}^{(t)}.$$

Section B. Fractionalization measure equivalency

The fractionalization measure used in the paper is:

$$F_p^{(t)} = 4\pi_p^{(t)}(1 - \pi_p^{(t)}).$$

Define fractionalization as one minus the sum of the squares of group shares:

$$\begin{aligned}
\tilde{F}_p^{(t)} &= 1 - \left[(\pi_p^{(t)})^2 + (1 - \pi_p^{(t)})^2\right] \\
&= 1 - \left[(\pi_p^{(t)})^2 + 1 - 2\pi_p^{(t)} + (\pi_p^{(t)})^2\right] \\
&= 1 - (\pi_p^{(t)})^2 - 1 + 2\pi_p^{(t)} - (\pi_p^{(t)})^2 \\
&= 2\pi_p^{(t)} - 2(\pi_p^{(t)})^2 \\
&= 2\pi_p^{(t)}(1 - \pi_p^{(t)})
\end{aligned}$$

So:

$$F_p^{(t)} = 2\tilde{F}_p^{(t)}.$$

Section C. Fractionalization and individual atypicality

Consider an individual i in party p at time t, with identity $x_i \in \{0, 1\}$. Define individual *atypicality* as:

$$\ell_i^{(t)} = 1 - \Pr(X = x_i \mid p, t).$$

This quantity is the probability that a randomly selected co-partisan differs from individual i on the identity dimension. Higher values indicate that the individual is less typical of their party.

If $x_i = 1$, then:

$$\ell_i^{(t)} = 1 - \Pr(X = 1 \mid p, t) = 1 - \pi_p^{(t)}.$$

If $x_i = 0$, then:

$$\begin{aligned}\ell_i^{(t)} &= 1 - \Pr(X = 0 \mid p, t) = 1 - [1 - \Pr(X = 1 \mid p, t)] \\ &= 1 - (1 - \pi_p^{(t)}) = \pi_p^{(t)}.\end{aligned}$$

Using the definition of expectation:

$$\begin{aligned}\mathbb{E}\left[\ell_i^{(t)} \mid p, t\right] &= (1 - \pi_p^{(t)}) \times \Pr(X = 1 \mid p, t) + \pi_p^{(t)} \times [1 - \Pr(X = 1 \mid p, t)] \\ &= (1 - \pi_p^{(t)})\pi_p^{(t)} + \pi_p^{(t)}(1 - \pi_p^{(t)}) \\ &= 2\pi_p^{(t)}(1 - \pi_p^{(t)})\end{aligned}$$

So, for each party p and time t:

$$F_p^{(t)} = 2 \times \mathbb{E}\left[\ell_i^{(t)} \mid p\right].$$

In other words, increasing fractionalization means that the average partisan becomes less typical of their own party.

Section D. Sorting and fractionalization

Let $0 < \epsilon < \frac{1}{5}$. At time $t = 0$, both parties contain a share ϵ of individuals with $X = 1$:

$$\pi_D^{(0)} = \epsilon, \qquad \pi_R^{(0)} = \epsilon.$$

Then sorting between parties is:

$$S^{(0)} = |\epsilon - \epsilon| = 0.$$

Fractionalization within each party is:

$$F_p^{(0)} = 4\epsilon(1 - \epsilon).$$

At time $t = 1$, let the population share of $X = 1$ increase, but its shares within the two parties increase with different speed:

$$\pi_D^{(1)} = 2\epsilon, \qquad \pi_R^{(1)} = 4\epsilon.$$

Sorting between parties becomes:

$$S^{(1)} = |2\epsilon - 4\epsilon| = 2\epsilon > S^{(0)}.$$

So, sorting between parties increases from $t = 0$ to $t = 1$.

Fractionalization within party D becomes:

$$\begin{aligned} F_D^{(1)} &= 4(2\epsilon)(1 - 2\epsilon) \\ &= 8\epsilon(1 - 2\epsilon). \end{aligned}$$

The change in fractionalization within party D is:

$$\begin{aligned} F_D^{(1)} - F_D^{(0)} &= 8\epsilon(1 - 2\epsilon) - 4\epsilon(1 - \epsilon) \\ &= 8\epsilon - 16\epsilon^2 - 4\epsilon + 4\epsilon^2 \\ &= 4\epsilon - 12\epsilon^2 \\ &= 4\epsilon(1 - 3\epsilon) > 0 \end{aligned}$$

So, fractionalization within party D increases from $t = 0$ to $t = 1$.

Fractionalization within party R becomes:

$$\begin{aligned} F_R^{(1)} &= 4(4\epsilon)(1-4\epsilon) \\ &= 16\epsilon(1-4\epsilon). \end{aligned}$$

The change in fractionalization within party R is:

$$\begin{aligned} F_R^{(1)} - F_R^{(0)} &= 16\epsilon(1-4\epsilon) - 4\epsilon(1-\epsilon) \\ &= 16\epsilon - 64\epsilon^2 - 4\epsilon + 4\epsilon^2 \\ &= 12\epsilon - 60\epsilon^2 \\ &= 12\epsilon(1-5\epsilon) > 0 \end{aligned}$$

So, fractionalization within party R increases from $t = 0$ to $t = 1$.

The derivations above prove that there can exist two time periods $t \in \{0, 1\}$ such that:

$$S^{(1)} > S^{(0)} \quad \text{and} \quad F_p^{(1)} > F_p^{(0)} \text{ for } p \in \{D, R\}.$$

In other words, increase in between-party sorting and increase in fractionalization within both parties can indeed happen simultaneously.